OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4751

Introduction to Advanced Mathematics (C1)

Wednesday **12 JANUARY 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are **not** permitted to use a calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72.

WARNING

You are not allowed to use a calculator in this paper

This question paper consists of 4 printed pages.

Registered Charity 106696
[Turn over

Section A (36 marks)

1 Solve the inequality $2(x-3) < 6x+15$. [3]

2 Make r the subject of $V = \frac{4}{3}\pi r^3$. [3]

3 In each case, choose one of the statements

$$P \Rightarrow Q \qquad P \Leftarrow Q \qquad P \Leftrightarrow Q$$

to describe the complete relationship between P and Q.

(i) For n an integer:

P: n is an even number
Q: n is a multiple of 4 [1]

(ii) For triangle ABC:

P: B is a right-angle
Q: $AB^2 + BC^2 = AC^2$ [1]

4 Find the coefficient of x^3 in the expansion of $(2+3x)^5$. [4]

5 Find the value of the following.

(i) $\left(\frac{1}{3}\right)^{-2}$ [2]

(ii) $16^{\frac{3}{4}}$ [2]

6 The line L is parallel to $y = -2x+1$ and passes through the point $(5, 2)$.

Find the coordinates of the points of intersection of L with the axes. [5]

7 Express $x^2 - 6x$ in the form $(x-a)^2 - b$.

Sketch the graph of $y = x^2 - 6x$, giving the coordinates of its minimum point and the intersections with the axes. [5]

8 Find, in the form $y = mx + c$, the equation of the line passing through A $(3, 7)$ and B $(5, -1)$.

Show that the midpoint of AB lies on the line $x + 2y = 10$. [5]

9 Simplify $(3+\sqrt{2})(3-\sqrt{2})$.

Express $\dfrac{1+\sqrt{2}}{3-\sqrt{2}}$ in the form $a + b\sqrt{2}$, where a and b are rational. [5]

Section B (36 marks)

10

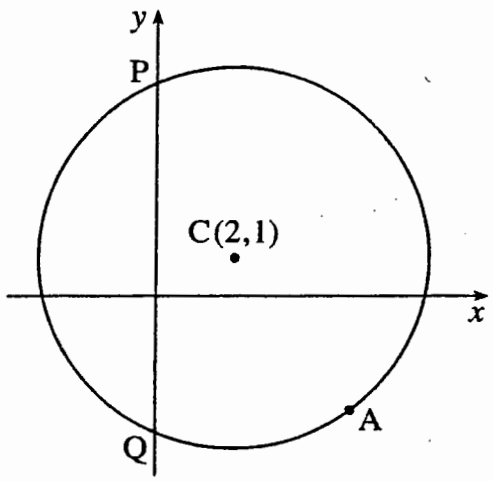

Fig. 10

Fig. 10 shows a circle with centre C (2, 1) and radius 5.

(i) Show that the equation of the circle may be written as

$$x^2 + y^2 - 4x - 2y - 20 = 0.$$ [3]

(ii) Find the coordinates of the points P and Q where the circle cuts the y-axis. Leave your answers in the form $a \pm \sqrt{b}$. [3]

(iii) Verify that the point A $(5, -3)$ lies on the circle.

Show that the tangent to the circle at A has equation $4y = 3x - 27$. [6]

11 A cubic polynomial is given by $f(x) = x^3 + x^2 - 10x + 8$.

(i) Show that $(x - 1)$ is a factor of $f(x)$.

Factorise $f(x)$ fully.

Sketch the graph of $y = f(x)$. [7]

(ii) The graph of $y = f(x)$ is translated by $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$.

Write down an equation for the resulting graph. You need not simplify your answer.

Find also the intercept on the y-axis of the resulting graph. [5]

[Turn over

12 (i) Show that the graph of $y = x^2 - 3x + 11$ is above the x-axis for all values of x. [3]

(ii) Find the set of values of x for which the graph of $y = 2x^2 + x - 10$ is above the x-axis. [4]

(iii) Find algebraically the coordinates of the points of intersection of the graphs of

$$y = x^2 - 3x + 11 \quad \text{and} \quad y = 2x^2 + x - 10.$$ [5]

Mark Scheme

OCR FINAL MARK SCHEME 4751 MEI PURE MATHS C1 JANUARY 2005

Section A

1	$2x - 6 < 6x + 15$ or $-4x < 21$ $-21 < 4x$ or ft $x > -21/4$ .o.e. (allow 21/–4 or better)	M1 M1 A1	condone ≤ for both Ms for inequality with +ve x coefft; if M0, SC1 for –21/4 found	3
2	$r = \sqrt[3]{\frac{3V}{4p}}$ o.e.	3	M2 for $r^3 = \frac{3V}{4p}$ o.e., with r^3 as subject, M1 for cube root of their r^3	3
3	(i) [P] ⇐ [Q] (ii) [P] ⇔ [Q]	1 1	condone Q ⇒ P; in both parts, condone arrows not implication symbols	2
4	1080 [x^3]	4	M1 for each of 2^2 and 3^3 or $(3x)^3$, and M1 for 10 or (5×4×3)/(3×2×1) or for 1 5 10 10 5 1 seen but not for $^5C_3$	4
5	(i) 9 (ii) 8 [condone –8 or ±8]	2 2	M1 for 3^2 or $(3/1)^2$ or $1/(1/3)^2$ M1 for $16^{\frac{1}{4}} = 2$; M0 for $\sqrt[4]{4096}$	4
6	$y = -2x + c$ $2 = -2\times5 + c$ or ft their gradient o.e. $c = 12$ (0, 12) or ft their line (6, 0) or ft their line	M1 M1 A1 1 1	or M1 gradient of $L = -2$ M1 for $x = 0$, $y = 2 - 5\times-2$ M1 for $y = 0$, $x = 5 - 2/(-2)$ no ft for $y = -2x + 1$ used or B5 for both correct answers; condone not given as coords if clear which axis	5
7	$a = 3$, $b = 9$ sketch of parabola correct way up min at (3, –9) or ft their $(x - 3)^2 - 9$ crossing x axis at 0 and 6	1+1 G1 G1 G1	or $(x - 3)^2 - 9$ seen isw correct shape, must extend above x axis may be stated elsewhere; need not be coords. may be stated elsewhere	5
8	$y = -4x + 19$ cao midpoint = (4, 3) verifying on line $x + 2y = 10$	3 1 1	M1 for $m = (-1-7)/(5-3)$ o.e. and M1 for $y - 7 =$ their m $(x - 3)$ o.e.	5
9	[9 – 2 =] 7 $\frac{1+\sqrt{2}}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$ $= \frac{3+2+3\sqrt{2}+\sqrt{2}}{7 \text{ or f.t}}$ o.e. $= \frac{5}{7} + \frac{4}{7}\sqrt{2}$	1 M1 M2 A1	 dep on prev M; M1 if one error [1 out of 5 terms, or 1 out of 3 or 4 terms if collected] condone $\frac{5+4\sqrt{2}}{7}$, isw	5

Section B

10	**i**	$(x-2)^2 + (y-1)^2 = 5^2$ $x^2 - 4x + 4 + y^2 - 2y + 1 = 25$ or $x^2 - 4x + 4$ and $y^2 - 2y + 1$ seen	M2 1	M1 for one side correct; for backwards working: M1 for $(x-2)^2 + (y-1)^2$ seen, A1 correct completing of squares shown; A1 for $(x-2)^2 + (y-1)^2 = 5^2$ or M1 for quote of *fgc* formula, A1 for correct substn, A1 completion for *c*	3
	ii	$y^2 - 2y - 20 = 0$ $y = \frac{2 \pm \sqrt{4+80}}{2}$ $= 1 \pm \sqrt{21}$	M1 M1 A1	subst of $x = 0$ attempt at use of formula or completing square; dep on prev M1 Pythag method: M1 for obtaining $\sqrt{21}$, A1 for each *y* value SC2 for $x = 2 \pm \sqrt{24}$ or $2 \pm 2\sqrt{6}$ from use of $y = 0$	3
	iii	subst of (5, –3) in eqn for circle grad. of CA = *y* diff / *x* diff attempt = –4/3 o.e. grad of tgt = ¾ or ft –1/their grad. $y + 3 = \frac{3}{4}(x - 5)$ ft their grad $4y + 12 = 3x - 15$ or $y = \frac{3}{4}x - 27/4$ o.e. NB ans $4y = 3x - 27$ given	1 M1 A1 M1 M1 1	or showing AC = 5 or M1 for $x = \frac{4y+27}{3}$ or $y = \frac{3x-27}{4}$ M1 for subst in eqn for circle M1 expn with at most one error A1 correctly obtaining $x = 5$ or $y = -3$ as only root A1 double root so tgt	6
11	**i**	f(1) attempted 1+1–10+8=0 one of $(x + 4)$ and $(x - 2)$ found the other [if B0 then M1 for roots –4 and 2] sketch of cubic the correct way up all ints with axes marked,	M1 A1 B1 B2 G1 G1	or M1 long divn as far as $x^2 + kx$ or $(x-1)(x^2 + bx - 8)$ A2 for $x^2 + 2x - 8$ oe B2 for $(x + 4)(x - 2)$ [mixed methods: mark one or other to adv. of cand.] correct or ft from their factors	7
	ii	$(x + 3)^3 + (x + 3)^2 - 10(x + 3) + 8$ or $(x + 7)(x + 2)(x + 1)$ oe eg $x^3 + 10x^2 + 23x + 14$ 14, or ft from their eqn if M1 or more earned; [20 from f$(x - 3)$]	3 2	M2 for $y = f(x + 3)$ or attempt to subst $(x + 3)$ or intercepts –7, –2, –1 or M1 for $y = f(x - 3)$ or subst $(x - 3)$ or intercepts –1, 4, 5 M1 for subst $x = 0$ in their eqn	5
12	**i**	use of $b^2 - 4ac$ [may be in quad. formula] $= 9 - 44$ oe [negative] so no [real] roots [condone not showing a pos. value]	M1 A1 A1	or M1 for $(x - 3/2)^2 + k$ and M1 for $k = 11 - (\text{their } 3/2)^2$ [or M1 for $y' = 2x - 3$ and M1 use of $y' = 0$] and A1 for min $y = 35/4$ or showing min is +ve	3
	ii	$(2x + 5)(x - 2)$ [>0] 2 and –2.5 oe identified sketch of parabola $x > 2$ or $x < -2.5$	M1 A1 M1 A1	[M0 for formula] or B2 or algebraic argument or B2; both needed; B1 if '=' included	4
	iii	$x^2 - 3x + 11 = 2x^2 + x - 10$ [0 =] $x^2 + 4x - 21$ [0 =] $(x + 7)(x - 3)$ $x = 3$ or –7; $y = 11$ or 81	M1 M1 M1 A1 A1	or subtraction to eliminate *y* rearrange to 0; condone one error attempt to factorise or use formula or A1 for (3, 11) and A1 for (–7, 81); M0 A0 for trial and imp.	5

Examiner’s Report